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Orbit Determination Error Analysis

AUGUST 1963

Prepared by D. R. SPEECE

Telecommunications and Tracking Department

Prepared for COMMANDER SPACE SYSTEMS DIVISION

UNITED STATES AIR FORCE

Inglewood, California

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ORBIT DETERMINATION ERROR ANALYSIS

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AEROSPACE CORPORATION
El Segundo, California

Contract No. AF 04(695)-269

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UNITED STATES AIR FORCE
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Prepared by:
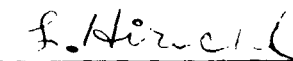
D. R. Speece
Analysis and Optimization
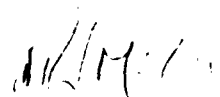
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ABSTRACT

The problem of orbit determination by radar tracking data is
broken into its constituent parts, which are then subjected to
detailed examination. Typical results are presented and dis-
cussed as a method of arriving at general conclusions. The
behavior of orbital errors is interpreted in terms of the influence
of their sources and the nature of their propagation. The em-
phasis of this analysis is on why orbital errors behave as they
do, how well can we expect to determine orbits, and how can
we do better. The conclusions are based on the results of a
large number of digital simulations with the Aerospace TRACE
program and the General Electric Pat-B program, plus limited
experience in the reduction of "live" data from the SCF net.

PREFACE

The Research reported herein was originally presented at the Eighth Symposium on Ballistic Missile and Space Technology, held at the U.S. Naval Training Center, San Diego, California, 16-18 October 1963.

CONTENTS

FIGURES

FIGURES (Continued)

ORBIT DETERMINATION ERROR ANALYSIS

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INTRODUCTION

The advent of large scale digital computer programs for the determination of the orbits of near-earth satellites has resulted in the need for criteria for the determination of the accuracy of the resultant orbits in areas remote from the tracking stations, and for techniques to improve this accuracy. This has lead, in turn, to the generation of large scale digital computer programs for the analysis of orbital errors. The results presented here were obtained by the use of such error analysis programs.

An orbital plane coordinate set is selected for the error analysis. The general characteristics and phase relationships of the position and velocity errors in this coordinate set are then presented. The dependence of the magnitudes of the orbital errors on the measurement system is presented in the form of bar charts of maximum position errors after one and two track-ing passes by systems which measured various quantities to vehicles in circular orbits at altitudes of 200 nm, 2000 nm, and 100,000 nm. Because these results represent the maximum capability in orbit determination when the only sources of error are station location errors, constant measurement biases, and Gaussian noise in each measurement, they are considerably better than can actually be achieved in most cases. Because of the interde-pendence of orbital errors through the equations of motion, the accuracy of determination of position relative to velocity at any point in orbit is not a function of whether the determination was by range or by range rate measure-ments. A curve of the equivalence of range and range rate as orbit determi-nation quantities is given. The advantages of interferometer systems in orbit determination is also discussed. In conclusion, an analysis of the fac-tors which limit the practical accuracies of orbit determination is given.

The orbit determination simulation programs take a specified orbit, earth model, and tracking system. The ensemble average errors in the system orbital prediction are then computed by statistical combination of all the tracking data. The resultant estimates are reliable only as long as the predicted orbital errors are larger than the uncertainties due to neglected sources of error.

In the radar tracking orbit determination problem, the sources of error are:

1. Radar measurement errors

2. Coordinate reference errors

3. Earth shape and mass uncertainties

4. Atmospheric drag uncertainty

5. Uncertainties in extraterrestrial perturbations

6. Uncertainties in reference metrics such as the radius of the earth, the speed of light, and the astronomical unit

7. Computational errors.

The first two of these are dependent on the basic capability of each radar tracker and the care with which it is installed, surveyed in, and calibrated. Station timing accuracy is dependent on the accuracy of reception and use of an external reference signal such as that provided by radio station WWV. The basic measurement accuracy is limited by target characteristics and by the accuracy of atmospheric corrections. The lower limit on the coordinate reference errors is imposed by area survey accuracy which is, in turn, ultimately limited by uncertainties in the shape and mass distribution of the earth.

With respect to mass perturbations by the geopotential harmonics and by the sun and moon, these are sources of orbital error only if they are omitted from the data reduction program. If the source of perturbation is included in the program model, then only the error in its computed vector magnitude is a source of orbital error.

The accurate tracking of many satellites over long periods of time has provided data for better estimates of the data reduction program constants such as the geopotential harmonic terms, solar radiation pressure, the structure of the atmosphere, the radius of the earth, etc. For near earth orbits, the uncertainties in the speed of light and in the astronomical unit are not significant enough sources of error to allow appreciable reduction in their uncertainties by tracking data reduction. It is important to recognize that the ability to calibrate the computer program earth model and to correct measurement bias and survey errors by tracking data reduction is strongly dependent on the relative contribution of the particular error source to the resultant total error in the computed orbit. Thus, the third zonal harmonic (pear shape) was found by its cumulative long-term effect on low inclination orbits. The resultant departure of the computed orbit from the measured orbit exceeded the limits assignable to measurement bias and survey error alone.

The orbit determination programs generally compute residuals between measured quantities and computed quantities. The nonlinear orbital equations are expanded in Taylor series about a nominal solution. The resultant linear set of first difference equations is then solved by a matrix formulation to obtain agreement between the computed orbit and the measured data. The criterion for satisfactory solution is usually minimization of the weighted sums of the squares of the residuals. An alternate weighting technique is the maximum likelihood criterion which provides optimum recognition of the effects of measurement bias and survey errors.

-2-

An obvious computational error in orbit determination is computer roundoff error. The above matrices tend to be ill-conditioned. Because of this, double precision computation of the terms in the matrices is often necessary. Even then, erratic results can often be traced to computer roundoff errors.

Let us now examine some of the characteristics of orbital errors. For our purposes the most convenient coordinate set is a vehicle centered set in which position and velocity components are measured in the direction of the horizontal component of the velocity vector (in-track), normal to the orbital plane (cross-track), and along the earth-central radius vector (altitude). As will be shown, this set has close physical coupling to the conventional orbital elements.

CUMULATIVE ERROR PROPAGATION

The orbital error analysis programs generally compute one-sigma ephemeris uncertainty matrices resulting from a tracking pass. These are analytically propagated along the orbit to the next tracking station. The new measurements result in reduced orbital errors which are then propagated to the next station. This process is continued over the period of interest for the problem at hand.

Typical behavior of the orbital errors is illustrated in figures 1a, 1b, and 1c. The general pattern of these results has been repeated in many simulations of the capabilities of a single tracking station which measures range, range rate, and/or angles once per revolution to a vehicle in a nearly circular orbit at any altitude. The curves present the components of ephemeris uncertainty at each point along the orbit as a result of errors in all preceding tracking data. The position and velocity errors are presented in pairs to emphasize the phase relationships between them. We will now investigate the reasons for this particular pattern.

Consider the behavior of the positional errors. The increase in in-track error after the first pass (figure 1a) is primarily due to period error. But it is also dependant on the uncertainty in eccentricity. Hence, it increases most rapidly on the side of the earth opposite the tracker. The composite curve has a steady rise due to period error on which is superimposed a nearly sinusoidal oscillation whose period is equal to the orbital period as a result of eccentricity error.

The cross-track error (figure 1b) depends primarily on the accuracy of determination of the orbital plane (inclination and longitude of the ascending node). Since the orbital plane intersects the center of the earth, the cross-track error drops to its minimum value at the tracker and at the point on the opposite side of the earth from the tracker. This second point is better known than the first (in real-time error propagation) because the second half of the tracking pass contributed to its determination. The variable component of cross-track error is a rectified sine-wave whose period is equal to the orbital period. The rectification is a result of the rms nature of the error computation.

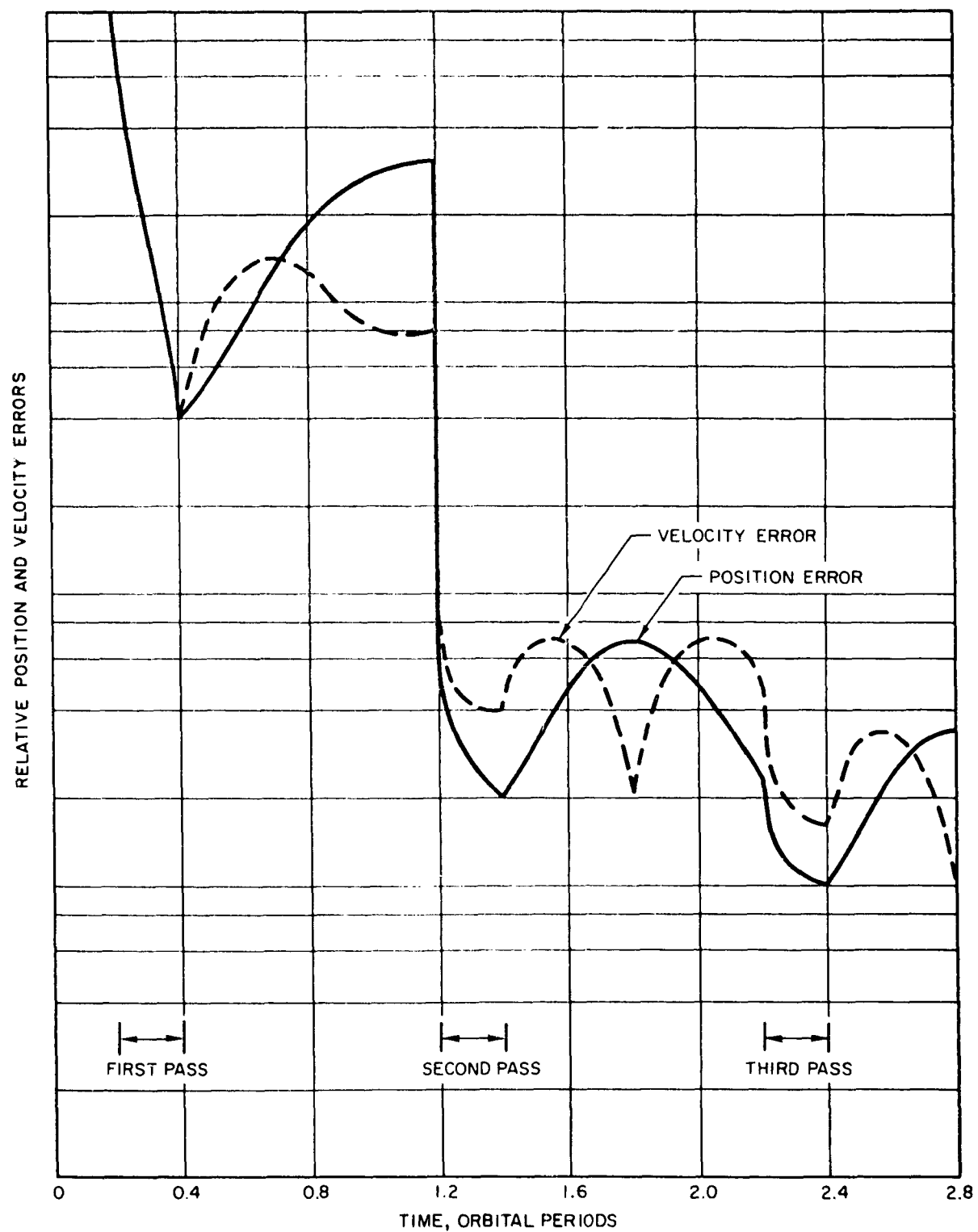Figure 1.   TYPICAL BEHAVIOR OF ORBITAL ERRORS

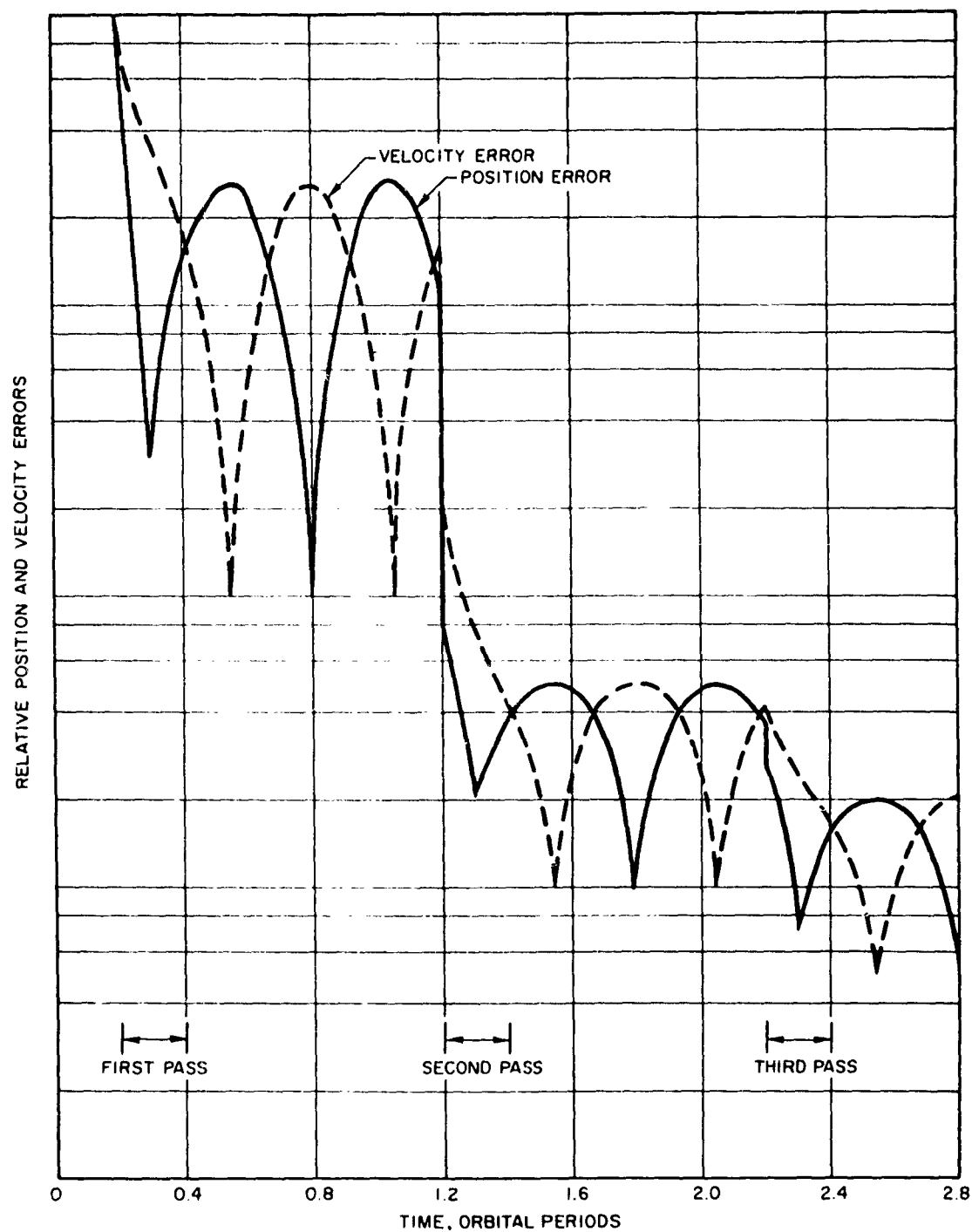1a.   IN-TRACK ERROR PROPAGATION

-4-

Figure 1. TYPICAL BEHAVIOR OF ORBITAL ERRORS
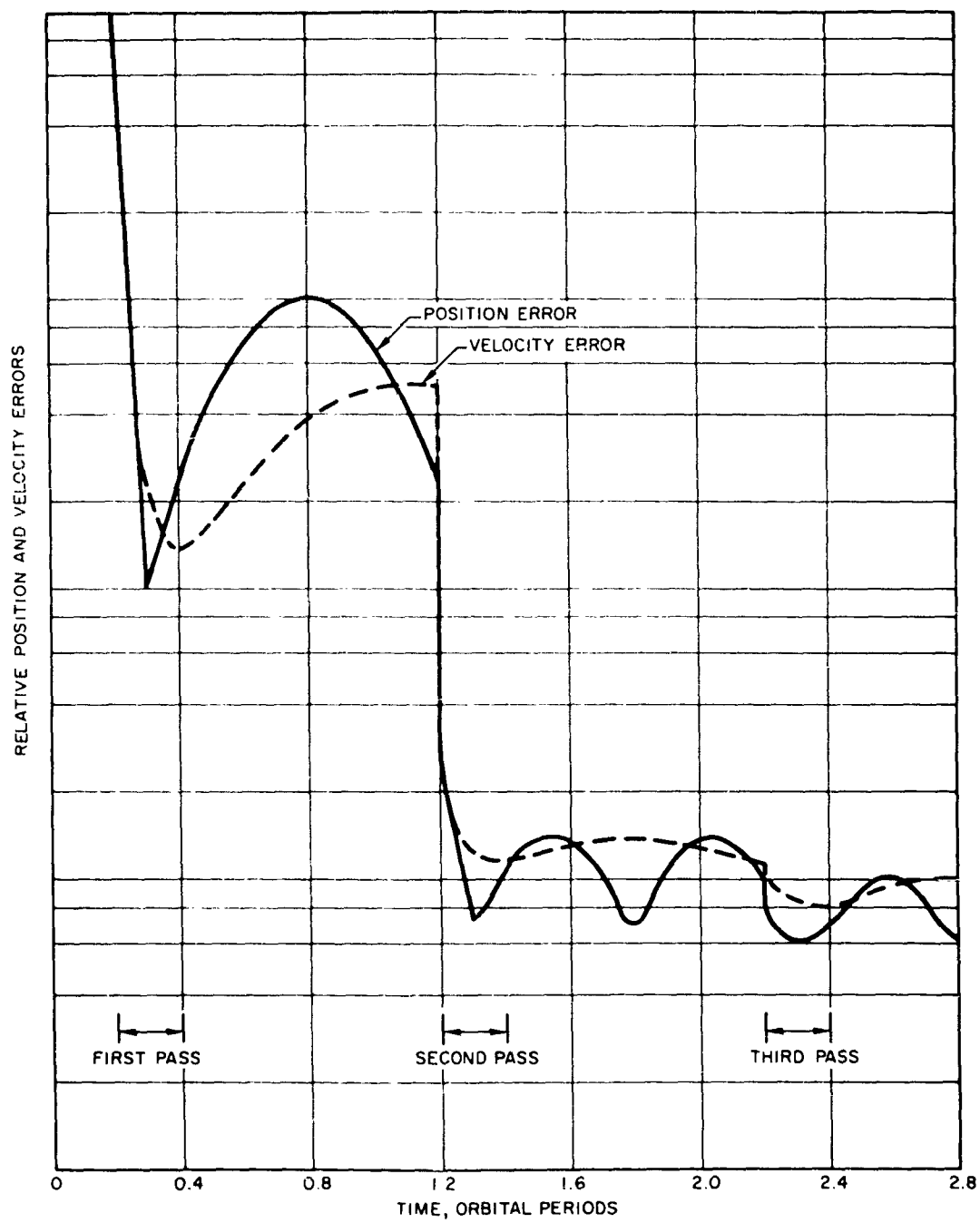
1b. CROSS-TRACK ERROR PROPAGATION

Figure 1. TYPICAL BEHAVIOR OF ORBITAL ERRORS

1c. ALTITUDE ERROR PROPAGATION

The altitude error following the first pass (figure 1c) is primarily a result of major axis uncertainty. Hence, it is roughly a half-sine wave between the first two tracking passes.

The nature of the error decrease during tracking can be explained as follows: at the minimum range from the tracker the slant range vector can be resolved into a cross-track component and an altitude component. The accuracy of the resolution is a function of the range and angle accuracy of the tracking system. The most valuable cross-track and altitude data is that obtained near the minimum range point. The best in-track data is often obtained near the radar horizon where the slant range vector is most nearly aligned with the vehicle velocity vector.

When the tracker first spots the vehicle on the second pass (roughly one revolution later) the in-track error is immediately reduced to a much lower level than before. The ballistic constraint (orbital equations of motion) allows a greatly improved estimate of orbital period. The resultant improvement in knowledge of the major axis of the orbit provides strong leverage in the computation of both altitude and in-track quantities. The eastward shift of the tracker due to earth rotation between passes provides a more accurate indication of the orientation of the orbital plane with a resultant reduction in cross-track errors.

After the second pass the in-track position error has the form of a half sine wave as would be expected if the tracker were near apogee or perigee of the orbit and the major axis were accurately known but eccentricity and the argument of perigee were still significantly in error. These same conditions explain the shape of the in-track velocity error and the altitude errors. Because of measurement errors, a tracker near perigee or apogee of an orbit invariably obtains a weaker solution for the argument of perigee (and path angle) than if the tracker were located elsewhere along the orbit. Because of the stronger solution for the path angle of the velocity vector, slightly eccentric orbits can apparently be more accurately determined than very nearly circular orbits. If the tracker is not located near apogee or perigee, then the phase of the eccentricity contribution to orbital error is shifted accordingly. It is then possible for the in-track and/or altitude position errors following the first tracking pass to continue to decrease for a short while after the end of tracking.

After two passes by the same tracking station, the orbit is often sufficiently well determined that measurement bias and station location errors result in relatively slow improvement with additional passes   However, because of some variability of physical system biases and because of the changes in the tracking geometry from pass to pass, there is some smoothing of the systematic measurement errors over succeeding passes. Also, the more accurate the measurement system, the less the relative improvement in the orbital estimate with a second pass. The effect of measurement type and accuracy is discussed later.

The velocity error curves bear a rough derivative relationship to the position error curves. This effect is most noticeable in the case of the cross-track errors. This is a result of the fact that the cross-track position

is most accurately determined at the tracker where the cross-track heading error in the velocity vector is a maximum. Because the orbital plane intersects the center of the earth, this results in errors of the following form:

$$\sigma_c \approx \sigma_{c_{max}} \left| \sin \frac{2\pi}{P} (t - t_o) \right| \quad ,$$

and

$$\sigma_{\dot{c}} \approx \frac{2\pi}{P} \sigma_{c_{max}} \left| \cos \frac{2\pi}{P} (t - t_o) \right| ,$$

where P is the orbital period and t_o is the time of the minimum slant range from the tracker. Because of the strong dependence of the first pass determination of orbital inclination on system angular (azimuth) accuracy, the maximum cross-track error after one tracking pass for low altitude orbits is

$$\sigma_{c_{max}} \text{ (first pass)} \approx r \bar{a} \quad ,$$

where r is earth central radius to the satellite and $\bar{a}$ is the smoothed azimuth error (usually the azimuth bias). As orbital altitude increases the longer tracking arcs permit the first pass determination of the orientation of the orbital plane by the station latitude dependent effect of earth rotation on range and/or range rate measurements. This effect is inclination dependent and vanishes for equatorial orbits.

The in-track and radial velocities for an elliptical orbit are of the form

$$\dot{I} = \frac{V_o}{\sqrt{1 - \epsilon^2}} (1 + \epsilon \cos \theta) \quad ,$$

and

$$\dot{A} = \frac{V_o}{\sqrt{1 - \epsilon^2}} \epsilon \sin \theta \quad ,$$

where V_o is the mean orbital velocity, ϵ is eccentricity, and θ is angle from perigee. Assuming small eccentricity, the errors in these are of the form

$$\Delta \dot{I} \approx \Delta V_o + V_o \left(\Delta \epsilon \cos \theta - \epsilon \Delta \theta \sin \theta \right) \ ,$$

and

$$\Delta \dot{A} \approx \epsilon \Delta V_o \sin \theta + V_o \left(\Delta \epsilon \sin \theta + \epsilon \Delta \theta \cos \theta \right) \ .$$

Between tracking periods, ΔV_o, $\Delta \epsilon$, and $\Delta \theta$ remain constant. After the orbital period has been accurately determined by a second tracking pass, the error in semi-major axis and, hence, V_o is negligible. We then have

$$\Delta \dot{I} \left(\theta - \frac{\pi}{2} \right) \approx \Delta \dot{A} \left(\theta \right) \ , \quad \Delta \dot{A} \left(\theta - \frac{\pi}{2} \right) \approx - \Delta \dot{I} \left(\theta \right)$$

The statistical combination of the above components of error and their integrals produce curves of the form of figures 1a and 1c. It has been found that there is strong quarter-orbit correlation of the type suggested above. But it has been observed in the computer results that the maximum value of $\sigma_{\dot{A}}$ is invariably larger than the maximum value of $\sigma_{\dot{I}}$.

After two tracking passes, the apparent form of the in-track and altitude velocity errors of figures 1a and 1c is

$$\sigma_{\dot{I}} \approx \sigma_{\dot{I}_{max}} \left| \sin \left(\frac{2 \pi (t - t_o)}{P} + \sigma_{\theta_1} \right) \right| \ ,$$

and

$$\sigma_{\dot{A}} \approx \sigma_{\dot{A}_{max}} \left| \cos \left(\frac{2 \pi (t - t_o)}{P} + \sigma_{\theta_2} \right) \right| \ ,$$

where P is again the orbital period, t_o is time of tracker passage, and the σ_{θ_i} are phase errors related to the uncertainty in the location of perigee.

Because the coordinate directions in the orbital plane set rotate with orbital motion, the position errors are related to the above velocity errors as follows

$$r_I \approx \frac{1}{2} \int_0^t \sigma_{\dot{I}}(t)\, dt - \frac{1}{2} \int_0^t \sigma_{\dot{A}}\left(t - \frac{P}{4}\right) dt$$

$$\approx \frac{P}{\pi} \left(\sigma_{\dot{I}_{max}} + \sigma_{\dot{A}_{max}} \right) \left[1 - \cos\left(\frac{2\pi(t - t_o)}{P} + \sigma_{\theta_3} \right) \right] ,$$

and

$$\sigma_A \approx \frac{1}{2} \int_0^t \sigma_{\dot{I}}\left(t - \frac{P}{4}\right) + \frac{1}{2} \int_0^t \sigma_{\dot{A}}(t)\, dt$$

$$\approx \frac{P}{\pi} \left| \left(-\sigma_{\dot{I}_{max}} + \sigma_{\dot{A}_{max}} \right) \sin\left(\frac{2\pi(t - t_o)}{P} + \sigma_{\theta_4} \right) \right| .$$

The factors $1/2$ and the cross-coupling with a quarter-orbit phase shift are in recognition of the interchange of coordinate directions each quarter orbit as a result of coordinate rotation. The signs of the integrated terms and the algebraic rather than rms addition are a result of the statistical correlations (indicated by the computer programs) between the components of error. The resultant expressions are seen to satisfy the observed fact that the maximum altitude position error is invariably considerably smaller than the maximum in-track position error. This appears to be a necessary result of the effect of the eccentricity error.

The above formulas have been derived in an attempt to explain observed results given by the error analysis programs. Rearranging them, we find

$$\sigma_{\dot{I}_{max}} \approx \frac{\pi}{P} \left| \sigma_{I_{max}} - 2\sigma_{A_{max}} \right| .$$

and

$$\sigma_{\dot{A}_{max}} \approx \frac{\pi}{P} \left| \sigma_{I_{max}} + 2\sigma_{A_{max}} \right| .$$

These approximations have been found to be very nearly correct for a wide variety of orbits. Although the altitude position error is smaller than the in-track position error, the converse is true of the corresponding velocity

-10-

errors. Physically, the reason can be traced to the large effect of the uncertainty in perigee location on the altitude rate error. Because orbital altitude at the tracking station is accurately determined, this uncertainty has a much smaller effect on the altitude position error.

Because the errors of figures 1a, 1b, and 1c represent average rms values, the basic sinusoidal oscillations are rectified and the effect of phase uncertainty and constant components is to fatten the curves and fill in the low points. The indicated phase relationships and the above ratios of velocity to position error are strong functions of the orbit determination geometry and are only weakly influenced by the quantities measured for orbit determination. But, as discussed in the next section, the actual magnitudes of the orbital errors are a strong function of the measurement mix and accuracy as well as orbital altitude.

THE CAPABILITIES OF VARIOUS MEASUREMENT SYSTEMS

Let us examine the orbital accuracy obtainable from the data of a single tracking station which measures various quantities to various accuracies. For the sake of uniformity, let the tracker be located at VAFB (34.7 deg N Latitude, 120.6 deg W Longitude), have a data rate of one set of measurements per 20 sec, a station location uncertainty of 100 ft N-S, 100 ft E-W, and 50 ft vertically, and limit the tracking to elevation angles above 10 deg. Assume the only sources of orbital error are the above survey errors plus white Gaussian noise and constant biases in each measurement. Assume data reduction is by the maximum likelihood criterion

The results of the system comparison are given in figures 2, 3, and 4 which present the maximum one-sigma positional errors after one and two tracking passes for various measurement accuracies and orbital altitudes. The altitudes selected were 200 nm, 2000 nm, and 100,000 nm. The 200 nm orbit had an inclination of 40 deg. The satellite passed to the east of the tracker at a maximum elevation angle of 45 degrees on the first pass and to the west of the tracker at a maximum elevation angle of 45 deg on the second pass. The 2000 nm orbit was polar and also had maximum tracking elevation angles of 45 deg for the first two passes. The 100,000 nm orbit was equatorial and, hence, had a maximum tracking elevation angle of 54.1 deg

Figure 2a compares the maximum ephemeris prediction errors of various systems after tracking a satellite for one pass in a 200 nm orbit In each case the system which measured range rate (0.3 fps bias, 0.03 fps random) produced more accurate estimates than the system which measured range (300 ft bias, 300 ft random). As can be seen, the cross-track error increases almost linearly with angular (azimuth) bias.

Figure 2b illustrates the order of magnitude improvement in ephemeris prediction after a second tracking pass. The range rate results are now seen to be almost independent of angular accuracy. Fits to low altitude SCF net range and angular data have shown a similar lack of sensitivity of multiple pass fits to the omission or inclusion of angular measurements But in these cases more than one station and more than two passes were used Also, at the 200 nm altitude, the three per minute data rate of figure 2 is insufficient for good smoothing of the assumed 300 ft random range error.
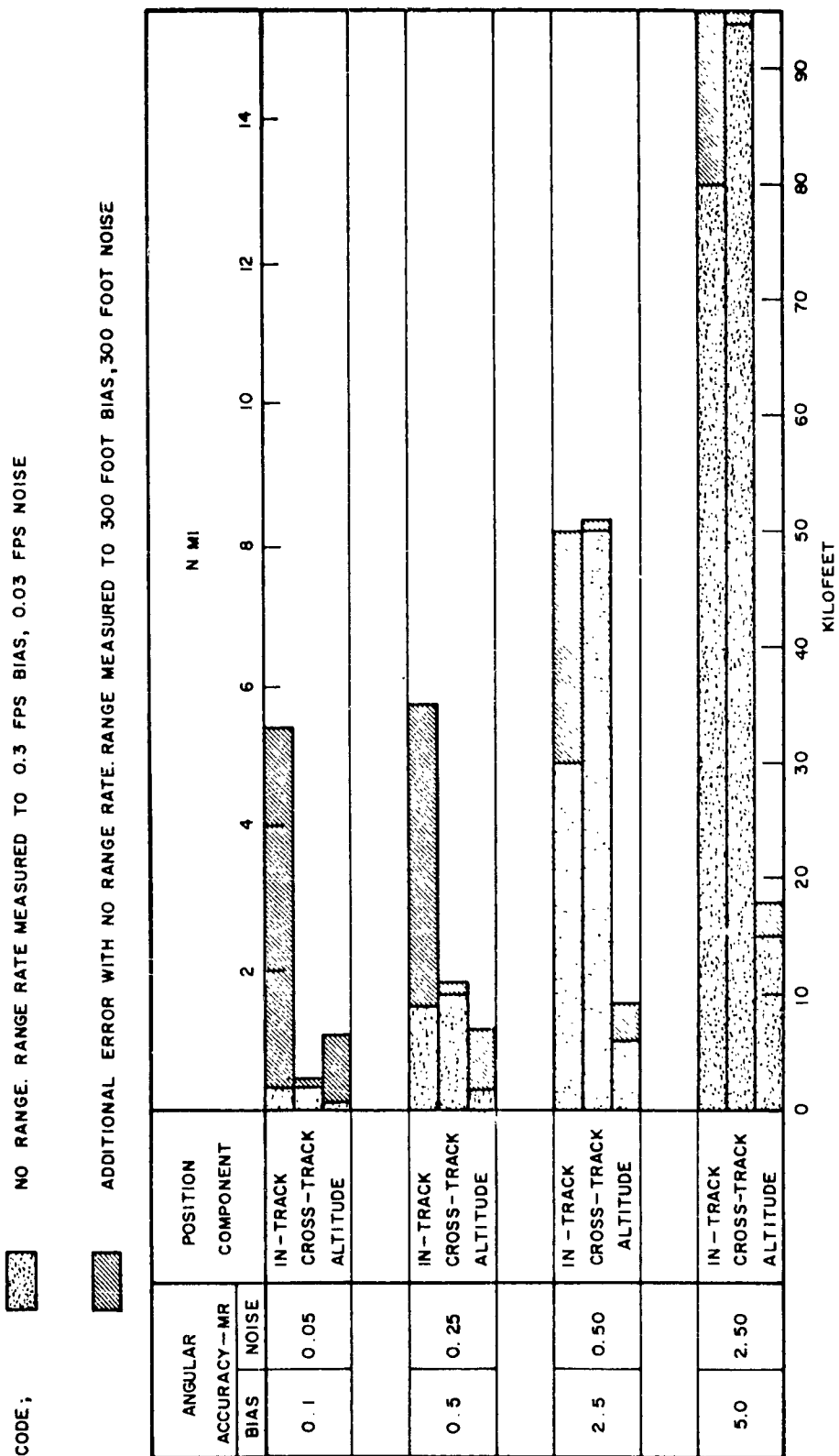
Figure 2. ONE-SIGMA POSITION ERRORS FOR 200 NM CIRCULAR ORBIT
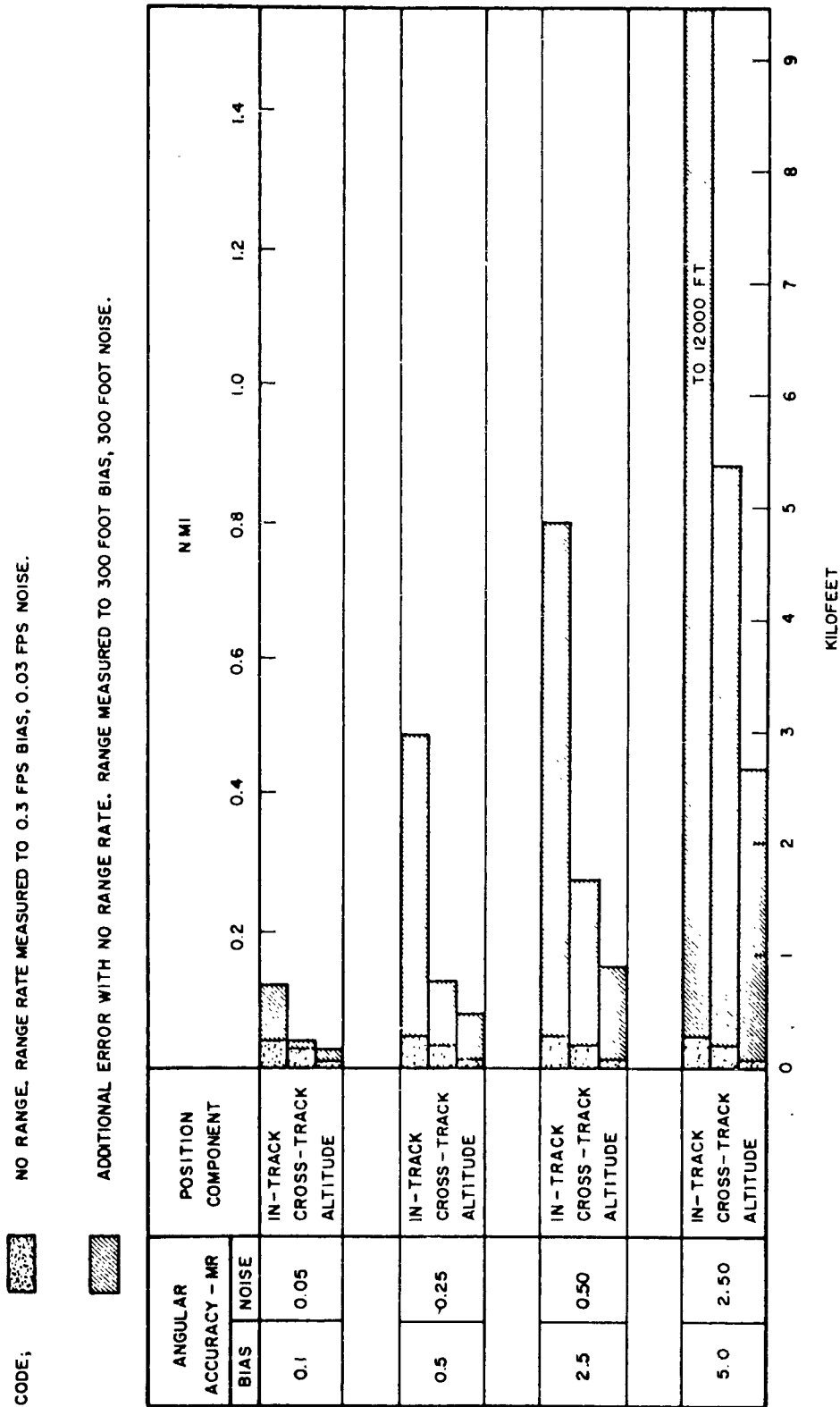2a. MAXIMUM ERRORS AFTER FIRST PASS

Figure 2. ONE-SIGMA POSITION ERRORS FOR 200 NM CIRCULAR ORBIT
2b. MAXIMUM ERRORS AFTER SECOND PASS

Figure 3. ONE-SIGMA POSITION ERRORS FOR 2000 NM CIRCULAR ORBIT

3a. MAXIMUM ERRORS AFTER FIRST PASS

CODE; ▨ RANGE MEASURED TO 35 FOOT BIAS, 5 FOOT NOISE

▨ NO RANGE MEASUREMENT

N MI

0.2 0.4 0.6 0.8 1.0 1.2 1.4

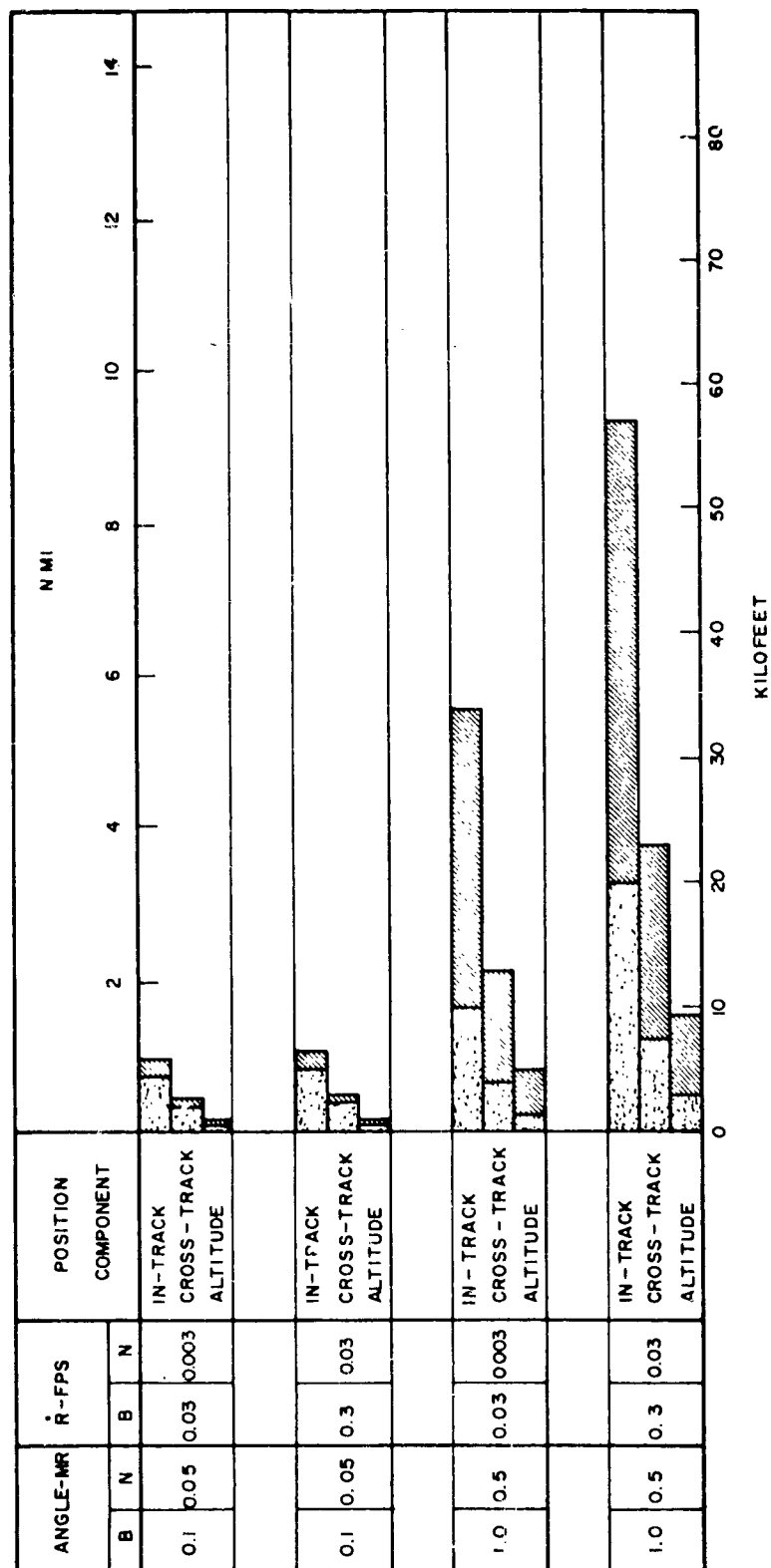| ANGLE-MR		Ṙ-FPS		POSITION COMPONENT
B	N	B	N	
0.1	0.05	0.03	0.003	IN-TRACK / CROSS-TRACK / ALTITUDE
0.1	0.05	0.3	0.03	IN-TRACK / CROSS-TRACK / ALTITUDE
1.0	0.5	0.03	0.003	IN-TRACK / CROSS-TRACK / ALTITUDE
1.0	0.5	0.3	0.03	IN-TRACK / CROSS-TRACK / ALTITUDE

0 1 2 3 4 5 6 7 8

KILOFEET
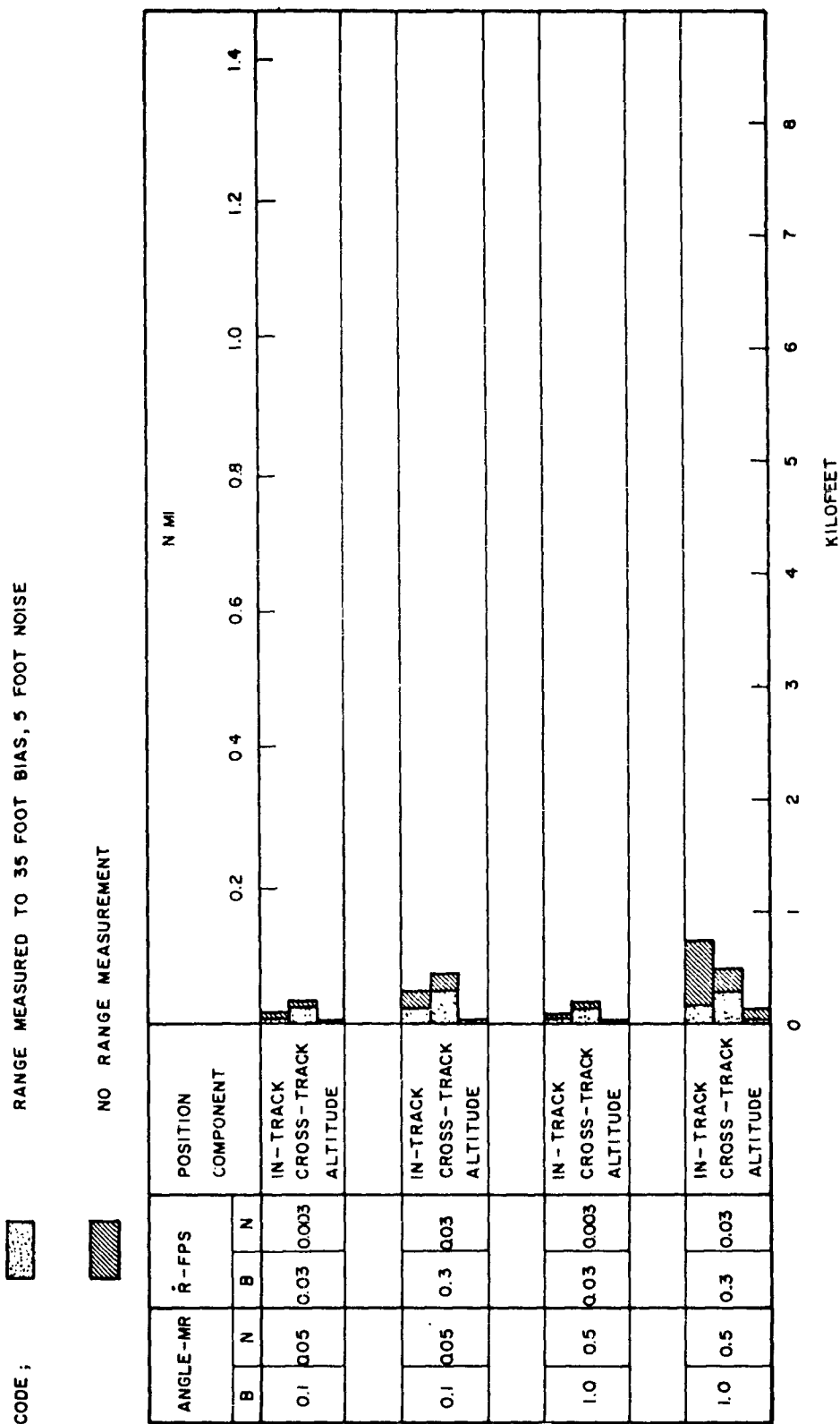
Figure 3. ONE-SIGMA POSITION ERRORS FOR 2000 NM CIRCULAR ORBIT

3b. MAXIMUM ERRORS AFTER SECOND PASS

ONE SIGMA MEASUREMENT ACCURACY

	ANGLES – MR	RANGE – FT	R – FPS
BIAS	1.0	35	0.30
NOISE	0.5	5	0.30

NOTE;
THE 100,000 N MI ORBIT WAS EQUATORIAL AND WAS TRACKED A SECOND TIME
AFTER ONLY A SMALL PART OF A REVOLUTION

CODE;   [ ] WITH RANGE

[ ] NO RANGE MEASUREMENT

POSITION COMPONENT	ORBITAL ALTITUDE – N MI
IN – TRACK	200
	2,000
	100,000
CROSS – TRACK	200
	2,000
	100,000
ALTITUDE	200
	2,000
	100,000

N MI

KILOFEET

Figure 4a.   VARIATION WITH ALTITUDE OF ONE-SIGMA POSITION
ERROR AFTER FIRST PASS

-16-

ONE-SIGMA MEASUREMENT ACCURACY

	ANGLES-MR	RANGE-FT	Ṙ-FPS
BIAS	1.0	35	0.30
NOISE	0.5	5	0.03

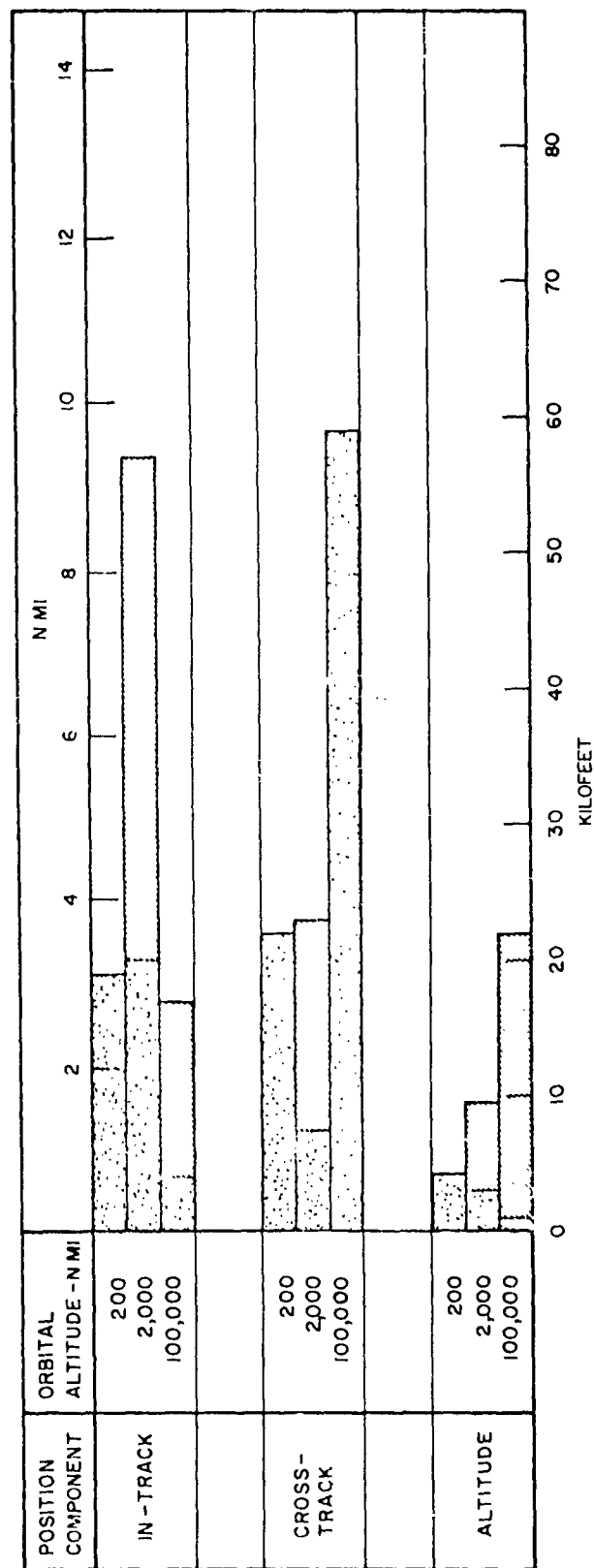NOTE: THE 100,000 N MI ORBIT WAS EQUATORIAL AND WAS TRACKED A SECOND TIME AFTER ONLY A SMALL PART OF A REVOLUTION

CODE: WITH RANGE

    NO RANGE MEASUREMENT

POSITION COMPONENT	ORBITAL ALTITUDE N MI
IN-TRACK	200
	2,000
	100,000
CROSS-TRACK	200
	2,000
	100,000
ALTITUDE	200
	2,000
	100,000

N MI

0.2   0.4   0.6   0.8   1.0   1.2   1.4

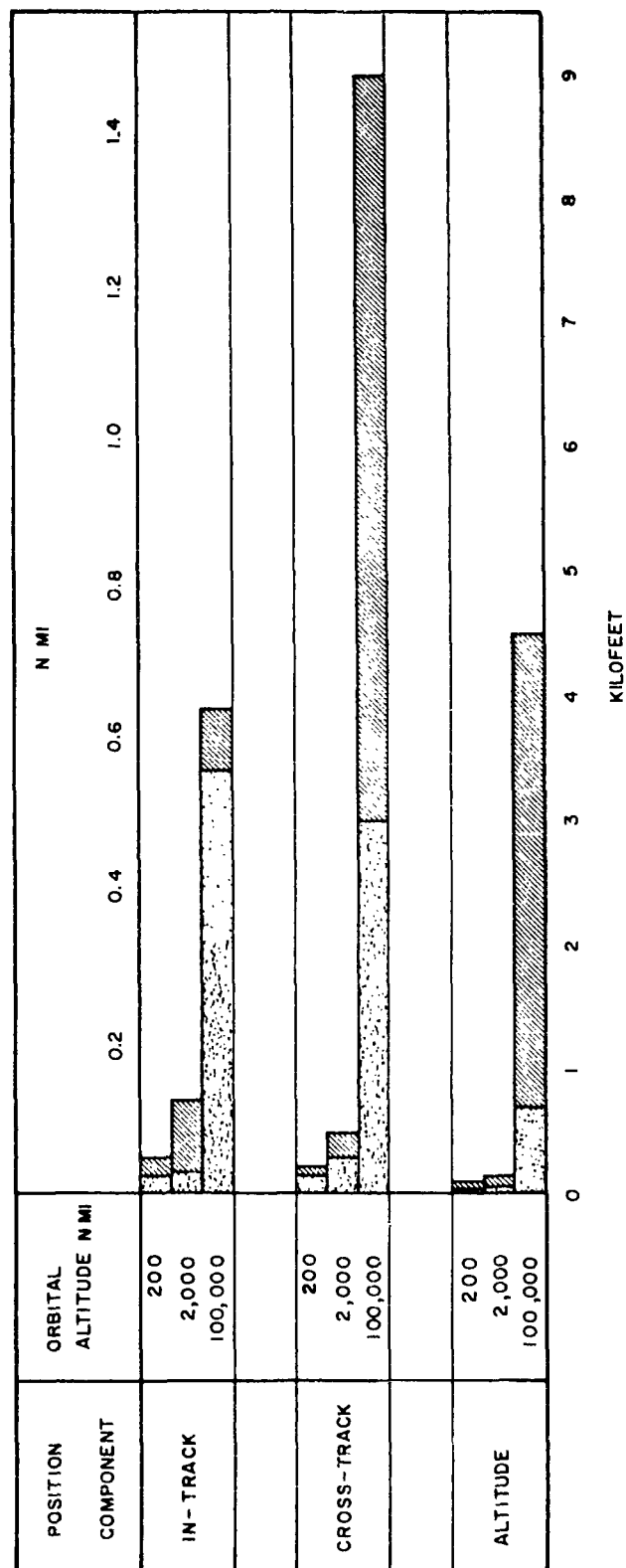KILOFEET

0   1   2   3   4   5   6   7   8   9

Figure 4b.   VARIATION WITH ALTITUDE OF ONE-SIGMA POSITION
ERROR AFTER SECOND PASS

Figure 3 shows the first and second pass ephemeris errors for a 2000 nm orbit. The relative lack of sensitivity of second pass results to angular accuracy is again evident. Also, at the higher altitude, the first pass cross-track error no longer shows a linear dependence on azimuth bias. The enhanced value of first pass range data with poorer angular data is also evident.

Figure 4 shows the value of 35 ft range measurements as a function of orbital altitude. The time between passes increased with altitude as follows:

Altitude (nautical miles)	200	2000	100,000
Time between passes (hours)	1.5	2.2	15.8
Orbital Period (hours)	1.5	3.0	232

Thus, earth rotation brought the 100,000 nm satellite back above the 10 deg tracking horizon again in one-fifteenth its orbital period. Because of this, the maximum positional errors did not correspond to the peaks of the curves of figure 1 but were considerably smaller. Keeping this in mind, the relative degradation in positional knowledge with vehicle altitude is evident in the first pass results of figure 4a and is amplified in the second pass results of figure 4b. The first pass results for the 200 nm orbit show no improvement in accuracy with the addition of range measurements, whereas there is improvement after the second pass. This is a result of the inability of the data reduction process to smooth the 100 ft station location error until after a second pass.

The 100,000 nm results are not strictly comparable to the lower altitude results. The positional errors given in figures 4a and 4b were at less than one-tenth revolution after the end of tracking rather than representing maximum values over the following revolution. Because the orbit was equatorial, the first pass cross-track error was primarily dependent on the angular tracking history. There is a generally weak dependence of first pass range and/or range rate data on inclination errors for nominally equatorial orbits (the situation is analogous to the problem of obtaining a good solution for the path angle of the velocity vector for nominally circular orbits). The second pass occurred only a small part of a revolution after the first. The small improvement in the second pass in-track position error with the measurement of slant range reflects the greater ability of the maximum likelihood data reduction technique to smooth range rate and angular biases than to smooth the combined range bias and survey error for very high altitude orbits.

In the orbit determination problem there is a rough equivalence between positional and rate measurements. Smoothed range data can be differentiated to obtain range rate. With the orbital constraints, range rate can be integrated to obtain range. The better of the two measurements in a combined system may carry practically all the weight in the orbital prediction process.

Figure 5 presents a rough curve of the above equivalence after several tracking passes. It presents the range rate accuracy required to permit the
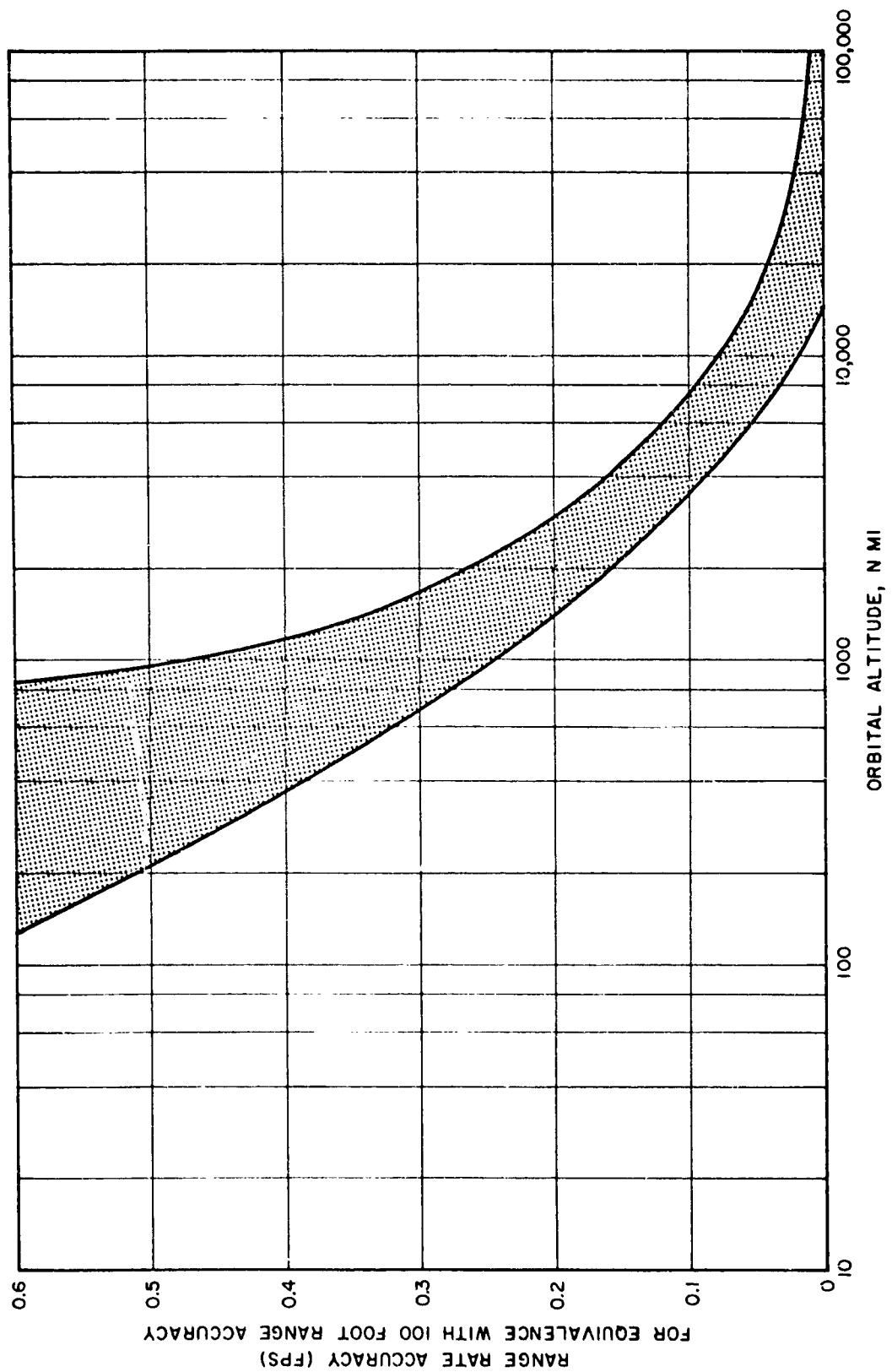
FIGURE 5. RELATIVE VALUE OF RANGE RATE DATA

calculation of ephemerides to the accuracy to which they can be computed from 100 ft range data. The spread in the curve represents uncertainty due to the small amount of data from which it was plotted plus the following factors:

1. Since the ability to deduce range from range rate tends to be inversely proportional to range, the low altitude results are strongly dependent on the maximum tracking elevation angles of the passes.

2. At high altitudes where the largest component of range rate is earth rotation, the results are strongly dependent on orbital inclination. Range rate has no capability in the determination of the orbit of a synchronous equatorial satellite.

3. The comparison related primarily to measurement biases but the range bias is augmented by station location errors.

4. The results are somewhat sensitive to the type of data reduction used.

The comparison would favor range more strongly after only one tracking pass. The greatly improved knowledge of the velocity vector by a second pass enhances the value of range rate data. Because of the strong bias smoothing effect of the maximum likelihood criterion, the results of figures 2, 3, and 4 could not be used in the plotting of figure 5.

A recognized technique for achieving the equivalent of good angular measurements is through the use of interferometer arrangements. Base legs can be angle surveyed to the accuracy of the local geodetic reference. Because of the elimination of common errors, differential quantities can be measured to at least an order of magnitude greater accuracy than the quantities being differenced.

The reduction in orbital position errors through the use of differential range rate data is illustrated in figure 6. The basic tracker was the 0.3 fps bias, 0.03 fps noise range rate and 1 mr bias, 0.5 mr noise angular tracker of figure 4. Differential range rates ($\dot{P}$ and $\dot{Q}$) were measured to 0.001 fps bias, 0.001 fps noise with orthogonal 10, 000 ft rate legs. These are extremely good accuracies. But the same results would be obtained with longer rate legs and less accurate measurements.

The first pass cross-track error reduction of figure 6b indicates that, in conjunction with the orbital constraint, the 0.001 fps differential range rate measurements are roughly equivalent to 0.015 mr angular measurements. As shown by figures 6a and 6c the resultant precision in determination of the orientation of the orbital plane permitted an order of magnitude improvement in the determination of the in-plane orbital elements from the basic first pass range rate history. As can be seen, the differential range rate data provided little improvement in the orbital estimate after the orbital plane orientation had been determined by the second pass range rate history of the basic tracker. But orbital motion provided a base leg of one revolution (21, 500 nm) for the latter determination.
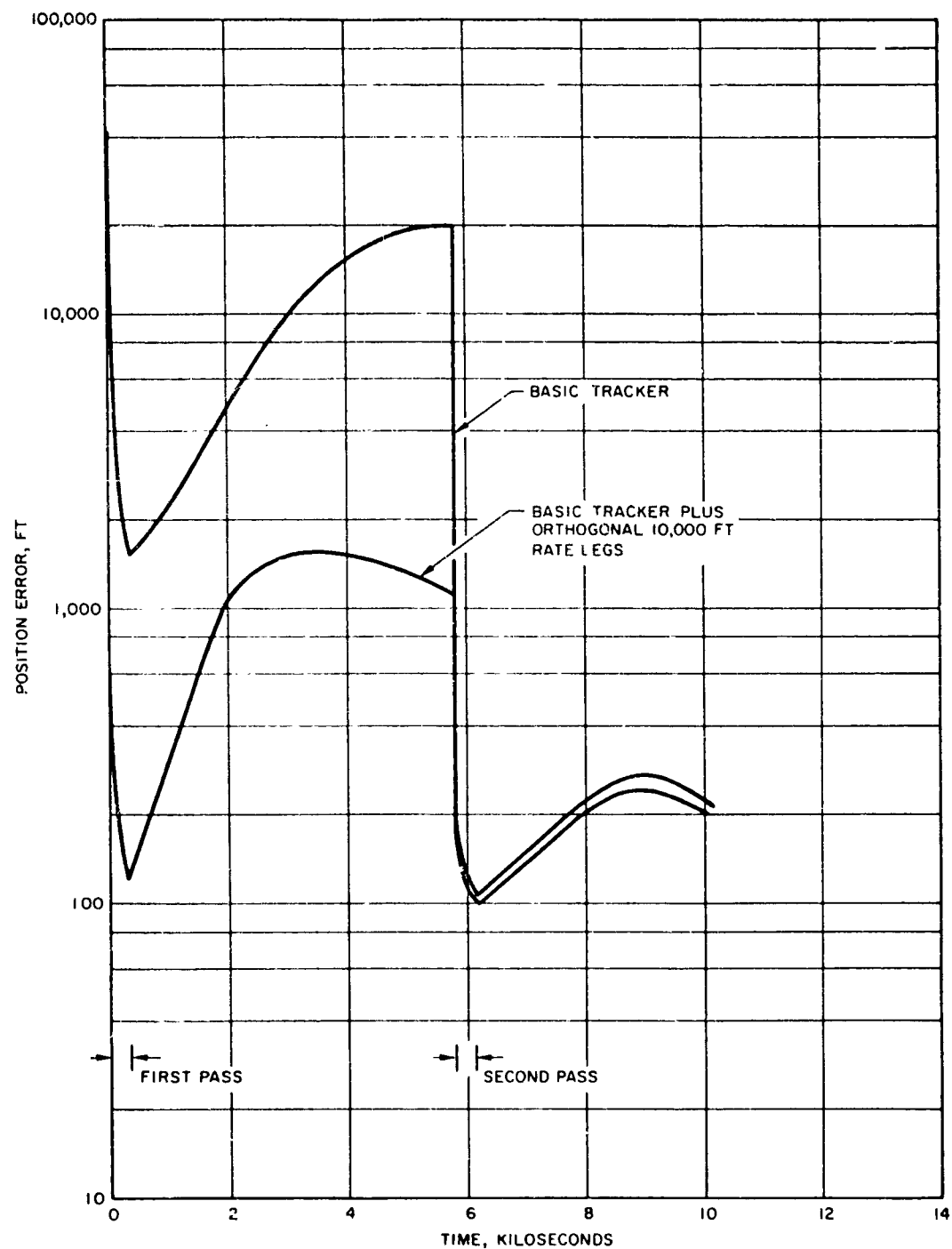
Figure 6. EPHEMERIS IMPROVEMENT WITH 0.001 FPS DIFFERENTIAL
RANGE RATE FOR 200 NM ORBIT

6a. ONE-SIGMA IN-TRACK ERROR

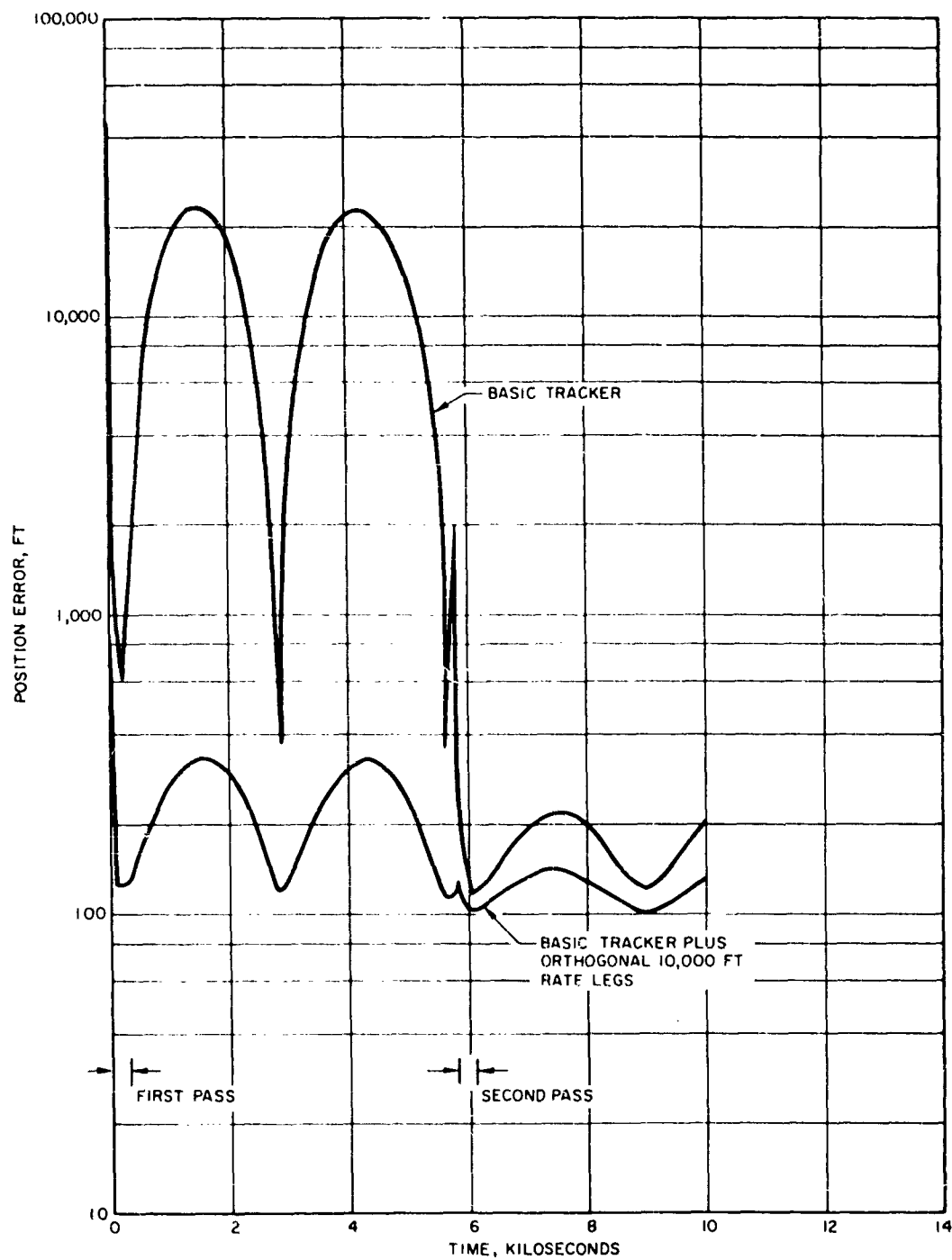Figure 6. EPHEMERIS IMPROVEMENT WITH 0.001 FPS DIFFERENTIAL
RANGE RATE FOR 200 NM ORBIT

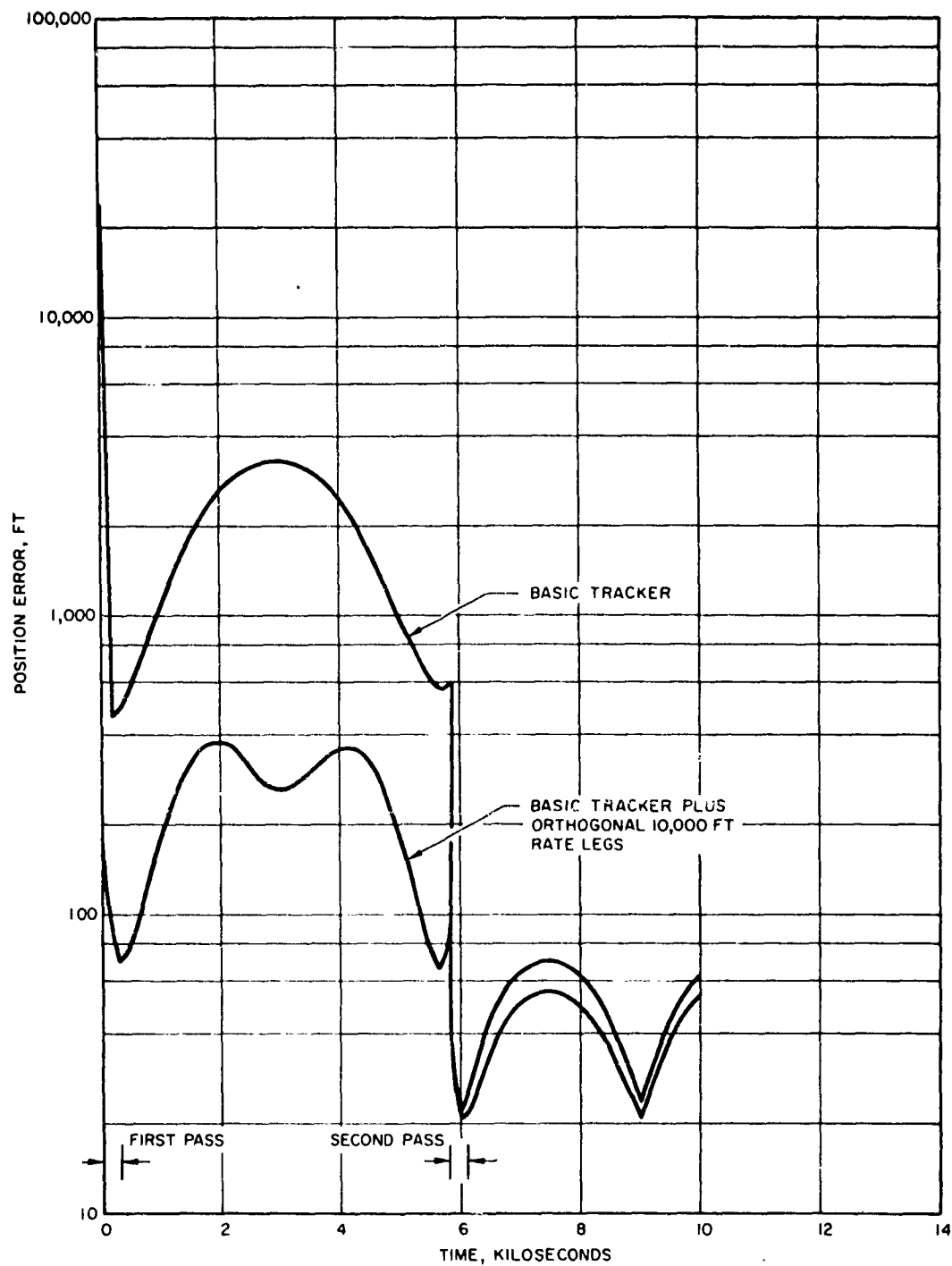6b. ONE-SIGMA CROSS-TRACK ERROR

-22-

Figure 6.   EPHEMERIS IMPROVEMENT WITH 0.001 FPS DIFFERENTIAL
RANGE RATE FOR 200 NM ORBIT

6c.   ONE-SIGMA ALTITUDE ERROR

It is important to recognize that the results given in figures 2, 3, 4, and 6 represent the capabilities of maximum likelihood data reduction with no earth model errors and with truly constant measurement biases. As discussed later, the same accuracy is obtained by differential correction of the measurement biases and station location errors. But the ability to obtain the accuracies of figures 2, 3, 4, and 6, particularly the results after two passes, depends strongly upon there being no sources of orbital error other than measurement noise, constant biases, and survey errors. Thus, these results represent a true upper limit to the capabilities of the systems discussed here. With optimum bias and survey correction, this limit is set by the random measurement errors. With inverse variance weighting of only the random errors and no attempt to correct bias and survey errors, the orbital accuracy after several passes is strongly limited by the bias and survey errors and is only a weak function of the random errors. In the actual orbit determination problem, the orbital accuracy obtainable eventually approaches a limit set by geopotential and atmospheric model errors.

THE EFFECTS OF MEASUREMENT GEOMETRY

The relative value of differential range rate and basic angular measurements deteriorates with orbital altitude in a manner similar to that depicted in figure 5 for range rate. The value of range and differential range as orbit determination parameters is enhanced at higher altitudes. The prime value of angular and differential quantities is in the first pass determination of the orientation of the orbital plane and in aiding convergence of the data reduction program.

Any six independent measurements are sufficient to determine the vehicle position and velocity at a point (an ephemeris). If position measurements are made at a point in orbit, only three of them can be independent. Any additional position measurements at this point are redundant and can be expressed as a function of any three independent position measurements. To estimate an ephemeris by measurement at a point, three independent rate measurements are also required. But time separation will usually assure independence even in repetitions of the same measurement. Thus, if a tracker makes accurate measurements at six points in time of R, A, E, $\dot{R}$, $\dot{P}$, $\dot{Q}$, P, or Q (rate measurements are insufficient for a circular synchronous equatorial satellite and an azimuth measurement or the equivalent is required by polar and equatorial trackers) then these six measurements are sufficient to determine an ephemeris. The ephemeris is obtained as that which results in an exact fit of the equations of motion to the six independent measurements. Subsequent measurements are then redundant and allow smoothing of the measurement errors.

If the orbit is nearly circular, then it becomes difficult for one tracker to separate a slight eccentricity effect from the inclination dependent rate of ground track departure from a great circle arc because of earth rotation. If only range and/or range rate is measured and the tracking time is short, then the single pass ephemeris estimate is subject to severe loss of precision. A second tracker greatly improves the situation regardless of what quantities are measured. The accuracy of the ephemeris estimate improves as the in-track separation of the two trackers is increased. With the orbital

-24-

constraint, this separation provides an effect analogous to that of base leg length in interferometer arrangements.

Examination of the shape of the positional error curves of figure 1 indicates that for the two station case, the optimum separation for the determination of the orientation of the orbital plane (the cross-track error) is 90 earth-central degrees along the orbital plane. A separation of 180 deg is optimum for the determination of the in-plane parameters. The best period estimate is obtained by the original station tracking at some multiple of 360 deg later (the higher the multiple, the more accurate the estimate for stable orbits). The use of the original station eliminates the prime effect of station location error.

The single pass range rate results for the 200 nm circular orbit showed a 30 to 1 degradation in in-track and altitude estimates and a 5 to 1 degradation in the cross-track estimate when the maximum tracking elevation angle was reduced from 83 deg to 9 deg with a 30 percent reduction in total tracking time. There is considerably less degradation if range rather than range rate is measured or if data from more than one pass is available. The above results are based on tracking above the radar 5 deg horizon at a data rate of one set of measurements per 5 sec.

The sensitivity of the ephemeris estimate to data rate and measurement noise was investigated. The prime effect of an increased data rate is greater smoothing of the noise. After a single pass the ephemeris accuracy is a strong function of measurement noise (and data rate). After more than one pass this dependence essentially vanishes (in the practical orbit determination problem) and the accuracy is a function of measurement bias, station location error, earth model error, and other sources of systematic error in either the data or the data reduction program model. After the orientation of the orbital plane is accurately determined by a second pass, the computer programs can apparently smooth the data noise much more effectively.

Interstation timing errors are of very little importance until they result in in-track displacements comparable to the interstation survey errors. For the low altitude orbits which present the worst case, a 4 millisecond station time reference error results in only a 100 ft in-track displacement. Another timing error can arise from the assignment of range and/or range rate measurement times to the time of ground reception rather than vehicle transmission of the signal. The speed of light is only 186 mi per millisecond. The result is an average time reference shift which is not important plus a variable bias in the data which is proportional to slant range differences and can be important.

The most serious kind of measurement bias has been found to be the constant bias. Variable components of bias are smoothed by the orbital equations after more than one pass in much the same way as data noise is smoothed. If the biases fluctuate from pass to pass then their effect averages out more rapidly than if they remain constant. The truly constant biases are smoothed only by changes in tracking history from pass to pass and by interstation averaging. The same is true of survey errors.

It has been found that if there is only one tracking station then the results are relatively insensitive to station location errors. The data gives no indication of a longitudinal survey error and is only weakly affected by a moderate error in radial distance of the station from the spin axis of the earth. Survey errors have an average effect which adds to the slant range bias plus a variable effect which is relatively less important. It can be argued that if survey errors are accounted for in a data reduction program then variable range biases should be ignored because of the poor ability of the program to differentiate between the two from their effects on the measurements.

In the multiple station case it has been found that survey errors have very little effect on range rate and angular measurements. It appears that survey errors are automatically smoothed in the reduction of range rate data. But they have an almost additive effect on slant range errors. Because of this, the most serious component of survey error is usually the interstation inconsistency rather than the uncertainty in location of the center of the earth.

DIFFERENTIAL CORRECTION OF SYSTEMATIC ERRORS

The data reduction program must obtain an estimate of the six ephemeris components at some point in time. For low altitude orbits it has been found very beneficial to also obtain an estimate of the average atmospheric drag. Station bias and survey errors can be treated as additional parameters and also be solved for. These estimates are then incorporated into the program as it re-solves for the six ephemeris components. This process of error correction by the data reduction programs has been called differential correction.

A problem in differential correction is the resultant increase in dimensionality of the error matrices with its attendant reduction in computing speed and increase in numerical accuracy problems. Maximum likelihood data reduction accomplishes the equivalent of differential correction by allowing the estimated orbit to adjust in a simple way to the systematic errors. A danger in either approach is that the error sources will be incorrectly identified.

The bias errors are identified as the average inconsistencies between the final computed orbit and the measurement sets. The station location errors are identified by their fixed directional characteristic. The resultant error evidences itself by a change in pattern between passes which bracket the tracking station. Range rate bias is indicated by the failure of the range rate null to occur at the minimum range point.

The sophisticated tracking data reduction programs presently available use the weighted least squares criterion for ephemeris fits. Without differential correction, this can result in ephemeris estimates which deteriorate when two stations which are close together track simultaneously. If there are range biases of opposite sign, for example, the combined least squares estimate can result in a large angular error in the assigned vehicle position.

The data presented earlier assumed maximum likelihood data reduction and no earth model errors. Similar results are obtained by the weighted least squares criterion with differential correction of the systematic errors provided convergence is satisfactory. The parameters to be included in the likelihood estimator or to be differentially corrected are the important sources of ephemeris error. There is very little ability to correct minor sources of error. Even the major sources of error cannot successfully be corrected until sufficient data is available to allow a reduction in ephemeris uncertainty to a level below the upper limit of the error in the quantity to be corrected. Differential correction cannot reduce measurement biases to a level below that of the systematic effect of the smoothed random errors.

In the determination of high altitude orbits from angular tracking data the quantities to be corrected are clearly the angular measurement biases. For the low altitude orbits it is difficult to assess the relative importances of earth model errors, drag uncertainty, measurement biases, and survey errors.

EARTH MODEL ERRORS

The prime component of the force field of near earth satellites is the inverse radius squared gravitational attraction toward the center of the earth. This effect is represented in the program model by values of earth mass and equatorial radius. The programs solve for satellite altitude above the surface of the earth. An error in the numerical value of earth mass and/or radius results in the following period error:

$$\Delta P = \frac{P}{2} \left(- \frac{\Delta \mu}{\mu} + 3 \frac{\Delta r_e}{r_e + h} \right) .$$

For a mass error of one part in 10^5 or a radius error of roughly 100 ft and a 2000 nm orbit, this results in a period error of 0.05 sec. Since average orbital periods have been measured by the Smithsonian system to at least an order of magnitude greater accuracy than this, we are forced to conclude that any residual errors in the present value of mass are largely balanced by compensating errors in earth radius.

In general, computer program model errors lead to orbital errors which are smaller than the positional differences which result from propagating in ephemeric with and without the source of model error. There are two reasons for this:

1. The data reflects the existence of the actual earth parameters and thus leads to a computed orbit which averages out part of the error.

2. Many of the model effects are oscillatory and a large part of the resultant error averages to zero over many sets of tracking data.

-27-

An earth mass or radius error can be compensated by assigning a corres-
ponding altitude error to the vehicle. It has been found that in the reduction
of range rate and angular data the programs readily convert earth mass and
station altitude errors to vehicle altitude errors, thus maintaining the meas-
ured tracking rates and orbital periods. If the orbital estimate is based on
slant range tracking histories, then this can no longer to done. Hence, slant
range trackers have a greater ability to differentially correct model errors
than other systems. But orbital estimates from range data without the cor-
rect computer program earth model parameters are poorer than estimates
from other measurements of comparable accuracy.

The actual mass distribution of the earth can be expanded in an infinite
series of spherical harmonics. If it were a rotating fluid mass of variable
density, then there would be no longitudinal harmonics and the geopotential
expansion would be

$$U = \frac{\mu}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{a}{r}\right)^n P_n (\sin \phi)\right]$$

where

μ = gravitational constant times earth's mass

r = earth central radius to point

J_n = mass coefficient of $n^{\underline{th}}$ harmonic

a = mean equatorial radius of earth

P_n = $n^{\underline{th}}$ zonal harmonic (Legendre polynomial)

ϕ = geocentric latitude to point .

This function is differentiated to obtain the components of gravitational force
at the point of interest (the satellite location).

By far the largest harmonic is oblateness (J_2). It has the effect of
precessing the orbital plane and the major axis. By measuring the resultant
long term precession rates, J_2 has been determined to an accuracy which is
undoubtedly much greater than the individual accuracies of the mass and
radius. The negative sign in the above equation is a result of the desire to
have a positive value for J_2.

Associated with J_2 is the shape oblateness (ϵ). Its computation from
J_2 requires the assumption of an internal density distribution for the earth.
An error in ϵ would appear as an altitude survey error which would increase
with distance of the station from the equator and would thus be primarily
deterimental to slant range data from far northern (or southern) stations.

The remaining zonal harmonics are roughly three orders of magnitude smaller than J_2 and their effects attenuate rapidly with orbital altitude. They have been isolated by their long term effects on low altitude, low inclination orbits.

Because of the irregular patterns of the continents and measured gravitational anomalies we would expect to also find evidence of longitudinal (tesseral) harmonics in the reduction of satellite tracking data. The geopotential expansion for these is

$$U = \frac{\mu}{r} \sum_{n=2}^{\infty} \sum_{m=1}^{n} J_{mn} \left(\frac{a}{r}\right)^n P_n^m (\sin \phi) \cos m(\lambda - \lambda mn)$$

where J_{mn} is the mass coefficient, P_n^m is the associated Legendre polynomial, and λ is longitude. These harmonics should have a maximum effect on low altitude polar orbits. One effect is an oscillatory orbital period. It has been found necessary to include J_{22} in the data reduction programs to obtain good fits to low altitude polar orbits. It has also been found that the best orbital prediction capability is obtained by fitting to tracking data taken over one-half day or one full day's time, thus obtaining optimum smoothing of the even tesseral harmonics. This also provides optimum smoothing of station location errors.

The orbit determination programs are forced to represent the effects of the full infinite series of geopotential harmonics by a few lower order terms. Because of the smoothing effect of the orbital constraint on local perturbations, this representation can be very good. But the results tend to be highly dependent on the inclination of the orbits used to compute the harmonics. At the present time, a set of harmonics has not been derived which will provide good fits to orbits of all inclinations. If properly interpreted, slant range data to low altitude nearly polar orbits should produce more valuable information on the longitudinal harmonics than any other data (provided drag is not a serious source of error).

The geopotential harmonic and station location errors can be handled by reducing enough different sets of data to arrive at good values for these parameters or by measuring range rate rather than slant range and thus obtaining a larger degree of smoothing of these effects. Slant range data can be converted to smoothed range rate data through the technique of one-pass fits.

In the low altitude region, drag is a major cause of ephemeris error. The atmospheric density is dependent on solar activity. Thus the 1959 ARDC standard atmosphere, which was derived from the behavior of satellites launched in 1958 (a year of high solar activity), indicated a mean density above 200 nm which is several times higher than that measured in 1962 (the year of the quiet sun). The density is higher on the daylight side of the earth.

-29-

In addition, solar radiation pressure shifts the center of the orbit away from the center of the earth toward the dark side. The radiation pressure is equal to drag at an altitude of roughly 500 nm.

The solution to the drag problem appears to be to use a sophisticated drag model with solar activity as an input and to have the data reduction program make early and repeated estimates of mean drag. Results with low altitude polar orbits suggest that the programs can estimate mean drag (the ballistic coefficient) accurately enough that, even with the ARDC 1959 atmosphere, the drag error may no longer be an important source of ephemeris error. With a more accurate set of geopotential harmonics and with good station locations this may cease to be true. At any rate, the change in drag as apogee altitude drops and as perigee shifts relative to the subsolar point should be correctly accounted for.

At lower altitudes the gravitational effects of external bodies can often be neglected. At higher altitudes drag and geopotential harmonics other than oblateness can often be neglected. Highly eccentric orbits with perigee in the drag region present an interesting case in which perturbations of perigee altitude by the sun and moon have a very great effect on orbital lifetime. For nearly circular orbits, the prime effect of the external bodies is to precess the orbital plane. If they are omitted then this precession will eventually exceed the limits of the ability of the computer program to improve its orbital estimate. As more data becomes available the ephemeris accuracy will initially improve, then level off, and eventually begin deteriorating. In the presence of any model error which has a cumulative effect, there is a maximum number of revolutions over which the computer program should obtain its fit. Beyond this point, earlier data must be dropped to maintain the accuracy of the estimate. This maximum number of revolutions is a measure of the accuracy of the computer program earth model. There is also a long-time limitation due to computer numerical accuracy.

CONCLUSIONS

It has been found that the basic changes in orbital uncertainty beyond a tracking station are a function of the equations of motion rather than measurement type or accuracy. The phase relationships and the ratio of position to-velocity error in each coordinate are dictated by the geometry and by the requirement that the position errors be the integral of the velocity errors rather than by whether the tracking system measured range or range rate. Only the scale magnitudes of the orbital errors are determined by measurement type and accuracy. The ideal minimum values of these numerical orbital errors for various systems whose only sources of error were station location uncertainty plus Gaussian noise and constant biases in each measurement are presented as bar charts of maximum positional errors after one and two tracking passes.

The results of the study indicate that for low altitude orbits after only one set of tracking data, system angular accuracy is of overriding important in ephemeris prediction. After the orbital plane is accurately determined by a second tracking pass using the same or a second station, angular data loses its value and good range rate becomes the most important measurement

for ephemeris prediction. At higher altitudes where range rate changes slowly, good slant range data becomes the most valuable.

After only one pass, ephemeris accuracy is strongly dependent on data rate and measurement noise. After more than one pass this dependence essentially vanishes and accuracy is limited by measurement bias and other systematic sources of error. In the multiple pass case, angular data should be de-emphasized or it will hurt the ephemeris estimates. But it should not be discarded because it improves program convergence.

In the low altitude case, the programs should make their own drag estimates after several revolutions. These should be repeated periodically. Station locations should also be determined by tracking data reduction. After this is done, errors in the geopotential harmonics may become the greatest remaining source of multiple pass prediction error. For high inclination orbits, the longitudinal harmonics have a significant effect. This effect and that of station location errors can be minimized by performing fits to data spans which are multiples of 24 hours.

In the presence of earth model errors the ephemeris estimate may deteriorate as the data span is increased. In addition, if tracking is concentrated along a small part of the orbit, the model errors will tend to convert to an eccentricity error rather than a period error. Hence, the data from one isolated station far from the others will cause an apparent deterioration of the fit.

Because the multiple pass estimate is only weakly affected by data noise and variable biases, the data sets can be truncated symmetrically about the minimum range points to increase the speed of the programs. In a simulation of a 5000 nm orbit, tracking above the 20 deg radar horizon produced the same ephemeris accuracy in three revolutions as tracking above the 5 deg radar horizon. In addition, actual data accuracy decreases at lower elevation angles and the errors tend to be symmetrical to either side of the maximum elevation angle point.

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Typical results are presented and discussed as a
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